

Cosmology with Modified Newtonian Dynamics (MOND)

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ABSTRACT

It is well-known that the application of Newtonian dynamics to an expanding spherical region leads to the correct relativistic expression (the Friedmann equation) for the evolution of the cosmic scale factor. Here, the cosmological implications of Milgrom's modified Newtonian dynamics (MOND) are considered by means of a similar procedure. Earlier work by Felten demonstrated that in a region dominated by modified dynamics, the expansion cannot be uniform (separations cannot be expressed in terms of a scale factor) and that any such region will eventually re-collapse regardless of the initial expansion velocity and mean density. Here I show that, because of the acceleration threshold for the MOND phenomenology, a region dominated by MOND will have a finite size which, in the early Universe ($z > 3$), is smaller than the horizon scale. Therefore, uniform expansion and homogeneity on horizon scale are consistent with MOND-dominated non-uniform expansion and the development of inhomogeneities on smaller scale. In the radiation-dominated era, the amplitude of MOND-induced inhomogeneities is much smaller than that implied by observations of the cosmic background radiation, and the thermal and dynamical history of the Universe is identical to that of the standard Big Bang. In particular, the standard results for primordial nucleosynthesis are retained. When matter first dominates the energy density of the Universe, the cosmology diverges from that of the standard model. Objects of galaxy mass are the first virialized objects to form (by $z=10$) and larger structure develops rapidly. At present, the Universe would be inhomogeneous out to a substantial fraction of the Hubble radius.

1. Introduction

The modified Newtonian dynamics (MOND) proposes that the law of inertia or gravity takes on a specific non-Newtonian form at accelerations well below a definite universal value (Milgrom 1983a,b,c). As an alternative to dark matter, the simple MOND formula with one new fixed parameter (the critical acceleration a_0) has been quite successful in predicting

the form of spiral galaxy rotation curves from the observed distribution of detectable matter (Begeman et al. 1990, Sanders 1996) as well as the magnitude of the conventional mass discrepancy in galaxy clusters (Milgrom 1983c) and in superclusters (Milgrom 1997). MOND subsumes the global scaling relationships for galaxies— the Tully-Fisher relation for spirals and the Faber-Jackson relation for ellipticals, as well as an equivalent gas temperature-mass relation for clusters of galaxies (Sanders 1994). MOND stabilizes rotationally supported thin disks (Brada 1996), explains the presence of a maximum critical surface density in spiral galaxies and ellipticals (Milgrom 1983b), and predicts the observed large conventional mass discrepancy in low-surface brightness systems (Milgrom 1983b, McGaugh & de Blok 1997).

However, an argument often directed against MOND is that, as a theory, it is ad hoc and incomplete. In particular, MOND makes no predictions with respect to cosmology and cosmogony. Even though the near coincidence of the empirically determined a_o with cH_o is suggestive of a cosmological basis for MOND, the structure of that cosmology is not at all evident. The naive expectation is that a hypothesis which posits such a radical departure from Newtonian dynamics (and presumably General Relativity) on the scale of galaxies and clusters of galaxies would surely lead to a highly unconventional cosmology which would be inconsistent with the phenomenological successes of the standard Big Bang— most notably the nucleosynthesis of the light elements in their observed abundances and the large and small scale isotropy of the Universe at the epoch of recombination. Such criticism cannot be addressed by an incomplete theory.

The reason for this incompleteness is that MOND at the present time lacks a relativistic extention; there is no credible general theory of gravity which predicts the MOND phenomenology in the appropriate limit. In fact, non-standard scalar-tensor theories have been proposed as a theoretical underpinning of MOND (Bekenstein and Milgrom 1984, Bekenstein 1988, Romatka 1992, Sanders 1997). Two of these, phase-coupling gravity (Bekenstein 1988) and stratified (preferred frame) theories with aquadratic scalar field Lagrangians (Bekenstein & Milgrom 1984, Sanders 1997) do yield sensible cosmologies— isotropic and homogeneous and similar to the low-density Friedmann models (Sanders 1989, Sanders 1997). Although such theories do have the considerable advantage of predictive power on scales other than extra-galactic, the fact is that they are unnatural; these non-standard scalar-tensor theories are as contrived as the ad hoc modification of Newton's laws which they presume to replace. Thus it is perhaps premature to work out fully the cosmological implications of such complicated and tentative theories.

Must, then, considerations of MOND cosmology then be postponed until the final theory is in place? Even before further theoretical developments, it may be possible to draw

some preliminary conclusions about a MOND universe by considering a finite expanding spherical region. It is well-known that the application of Newtonian dynamics to such a region leads to the Friedmann equation for the evolution of the cosmic scale factor. Even the cosmological constant can be included as an additional fluid with negative pressure. It might be expected that some insight into a pure MOND cosmology might be gained by such a procedure using Milgrom's formula instead of Newton's.

An interesting start in this direction was made by Felten (1984). He pointed out that with MOND, the physical size of the expanding region cannot be factored out. This means that uniform expansion of a spherical region is not possible; that the dynamical history of such a region in the Universe depends upon its physical size. This would suggest that an isotropic and homogeneous universe, as described by the Robertson-Walker (RW) metric is not possible in the context of MOND. Moreover, due to the effective logarithmic potential implied by MOND, any finite-size region will re-collapse in a finite time. The universe, out to the present horizon, will eventually re-collapse regardless of its mean density. In a low-density Universe a region with a present size of 20 to 30 Mpc would just now be turning around, and this, as stressed by Felten, is roughly the observed scale of large-scale structure— voids and superclusters.

In the present paper I continue the discussion of Felten on MOND cosmology in the context of finite expanding spherical regions. Three assumptions underly this discussion, all of which were more or less implicit in the work of Felten. The first is that the dynamics of such a region are not influenced by the exterior universe— that there is, in effect, an equivalent of the Birkhoff theorem for the relativistic theory of MOND. This assumption is the most shaky. Scalar-tensor theories of modified dynamics violate the strong equivalence principle which means that no dynamical system is isolated from its environment. The time-variation of the gravitational constant due to universal expansion is one example of the possible effect of the rest of the Universe on the spherical region. Here the assumption is that any such effects which may be present in the final theory will play a minor role in the dynamical history of the finite volume.

The second assumption is that the modified dynamics holds for all accelerations below a_o — that there is no return to Newtonian attraction or inertia at very much lower accelerations, as in cosmological PCG. This then will be an exploration of the cosmology of pure MOND.

Finally, it is assumed that the critical acceleration a_o is independent of cosmic time (not true in cosmological PCG). The fact that $a_o \approx cH_o$ would suggest that a_o is time dependent (as is the Hubble parameter). However, this expression could have one of several meanings as pointed out by Milgrom (1994). One possible basis is that $a_o \approx c\sqrt{\Lambda}$ where

$\Lambda \approx H_o^2$ is the cosmological constant. Such an interpretation would consistent with the assumption of no time variation of a_o . Alternatively, the numerical coincidence could arise from anthropic considerations: as will be shown below, structure develops when $cH \rightarrow a_o$.

Having made these assumptions, I re-derive Felten’s expression for the evolution of an isolated spherical region dominated by non-relativistic, pressure-less matter. I then demonstrate that, because of the acceleration threshold, modified dynamics can only be valid inside a critical radius r_c . This critical radius expands faster than the scale factor and the horizon, so that in the earlier universe the size of the region in which MOND applies is smaller than the horizon scale.

This is the principal result of the present paper. Friedmann cosmology, characterized by uniform expansion, applies on the scale of the horizon, while MOND, in which the expansion is highly non-uniform, applies on sub-horizon scales. Therefore, the usual Friedmann equation is valid for evolution of the universal scale factor, but the Felten equation is valid for spherical regions smaller than r_c . This means that, at any epoch, while the Universe overall is homogeneous, density inhomogeneities should be present on the scale of r_c at that time. Only recently in the history of the Universe has the size of the region dominated by MOND expanded to include a significant fraction of the the observable Universe, c/H_o (r_c at present depends upon the value of the cosmological constant); i.e., the Universe on large scale has only become “MONDIAN” at late cosmic time.

These results remain qualitatively the same when radiation is included. In the early universe, at the epoch of nucleosynthesis, MOND regions are very much smaller than the horizon. When pressure gradient forces are considered, the density and expansion of the universe remain highly uniform during the radiation-dominated era and identical to that of the standard hot Big Bang, so all results concerning primordial nucleosynthesis are retained. Moreover, MOND-induced inhomogeneities at the epoch of recombination are many orders of magnitude smaller than those implied by the observed fluctuations in the background radiation.

Regions of larger comoving size and mass enter the regime of modified dynamics at later times. Re-collapse of finite size regions dominated by modified dynamics proceeds rapidly once non-relativistic matter dominates the energy density, even in a very low density universe. This is due to the effective logarithmic gravitational potential implied by MOND. At the epoch of matter-radiation equality the mass enclosed within a MOND dominated region is $\approx 10^9 M_\odot$, comparable to a low mass galaxy. This gives a preferred mass scale to the first objects which collapse and virialize and may explain why galaxy mass objects are the fundamental virialized building blocks.

The expansion of MOND-dominated regions to include larger and larger comoving scales leads to a scenario of structure formation which is extremely hierarchical and “bottom up”, with the smallest objects forming first—star clusters and low mass galaxies—and larger structures forming later by a series of mergers. Massive galaxies should be in place as virialized objects by a redshift of 5 to 10. The largest objects just now being virialized are the rich clusters and supercluster scale objects which would only now be separating out of Hubble expansion. The scale-dependent deceleration induced by MOND implies that the Universe, at the present epoch, is inhomogeneous on large scale with a mean density of galaxies about a given galaxy decreasing out to hundreds of Mpc. The scale for cross-over to homogeneity depends on the value of the cosmological constant.

2. Dynamics of an Isolated Spherical Region

The Cosmological Principle which postulates the isotropy and homogeneity of the Universe permits separations between physical objects to be described by a universal dimensionless scale factor which is only a function of cosmic time. It is well-known (eg. see Peebles 1993) that the Friedmann equation for the evolution of scale factor can be derived by considering the Newtonian equation of motion for an isolated uniform spherical region of radius r :

$$\ddot{r} = -\frac{GM}{r^2}. \quad (1)$$

Here M is the active gravitational mass given, in the weak-field static limit of the Einstein field equations, by

$$M = \frac{4\pi r^3}{3}(\rho + 3p) \quad (2)$$

where ρ and p are the density and pressure of the smooth fluid. Combining eqs. 1 and 2 we have

$$\ddot{r} = -\frac{4\pi Gr}{3}(\rho + 3p) \quad (3)$$

which is supplemented by the energy conservation equation

$$\frac{d\rho}{(\rho + p)} = -\frac{3dr}{r} \quad (4)$$

and an equation of state. Obviously one may write $r = Lx$ where L is a fixed length scale and x is a time-dependent scale factor (here normalized to be 1 at the present epoch). Then L disappears in eq. 3 and 4. Taking the fluid to be a mixture of non-relativistic pressure-less matter ($p = 0$), radiation ($p_r = \frac{1}{3}\rho_r$), and vacuum energy density

$(p_v = -\rho_v = -3\lambda H_o^2/8\pi G)$ and integrating eq. 3 we find the usual dimensionless Friedmann equation for the time-evolution of the scale factor,

$$h^2 = \left(\frac{\dot{x}}{x}\right)^2 = \Omega_o x^{-3} + \Omega_r x^{-4} - (\Omega_o + \Omega_r + \lambda - 1)x^{-2} + \lambda \quad (5)$$

where λ is the dimensionless cosmological constant,

$$\Omega_o = \frac{8\pi G \rho_o}{3H_o^2} \quad (6)$$

is the density parameter for non-relativistic matter (with ρ_o being the present mean density of matter) and

$$\Omega_r = \frac{8\pi G a T_o^2}{3H_o^2 c^2} \quad (7)$$

is the density parameter for the cosmic background radiation where T_o is the temperature of the cosmic blackbody radiation at the present epoch (2.73 K) and a is the radiation density constant. The quantity $(\Omega_o + \Omega_r + \lambda - 1)$ is the integration constant which is to be identified with curvature of space-time. The quantity h is the Hubble parameter in units of the present Hubble parameter H_o and time is in units of the Hubble time H_o^{-1} .

MOND posits that for accelerations below a critical value a_o , the true gravitational force g is related to the Newtonian gravitation force g_n as

$$\mathbf{g}\mu(g/a_o) = \mathbf{g}_n \quad (8)$$

(Milgrom 1983a) where $\mu(x)$ is an unspecified function such that $\mu(x) \rightarrow 1$ if $x \gg 1$ and $\mu(x) = x$ if $x \ll 1$. Thus in the high acceleration limit the gravitational force is the usual Newtonian force, but in the low acceleration limit $g = \sqrt{g_n a_o}$ (this may also be written as a modification of the law of inertia where $F = m a \mu(a/a_o)$ replaces the usual expression). Because we are interested here only in a broad view of the overall dynamics of a MOND universe, we will assume that the transition in $\mu(x)$ between the two asymptotic limits occurs abruptly at $x=1$.

In the low acceleration limit the MOND equivalent of eq. 3 becomes

$$\ddot{r} = -\left[\frac{4\pi G a_o}{3}(\rho + 3p)r\right]^{\frac{1}{2}} \quad (9)$$

where now, obviously, a constant length scale cannot be factored out. Neglecting for the moment radiation and vacuum energy density and taking the equation of state only for non-relativistic pressure-less matter, the conservation equation implies that

$$\rho = \rho_o(r/r_o)^{-3} \quad (10)$$

where r_o is the comoving radius of the spherical region (i.e., the radius the spherical region would have at present if it continued to expand according to eq. 5) and ρ_o is the present mean density in the equivalent Friedmann model universe. Then eq. 9 becomes

$$\ddot{r} = -\left[\frac{\Omega_o}{2} H_o^2 r_o^3 a_o\right]^{\frac{1}{2}} r^{-1}. \quad (11)$$

This equation may be integrated once to give the equivalent of the Friedmann equation

$$\dot{r}^2 = u_i^2 - \left[2\Omega_o H_o^2 r_o^3 a_o\right]^{\frac{1}{2}} \ln(r/r_i) \quad (12)$$

where r_i is an initial radius of the sphere and u_i is the expansion velocity at this initial radius. From the form of eq. 12 it is obvious that at some maximum radius r_m the expansion will stop and the spherical region will re-collapse. This is given by

$$r_m/r_i = e^{q^2} \quad (13a)$$

where

$$q^2 = \frac{u_i^2}{(2\Omega_o H_o^2 r_o^3 a_o)^{\frac{1}{2}}} \quad (13b).$$

This is the expression derived by Felten (1984) written in a somewhat different form. At first sight, it may seem odd to use terms such as Ω_o which are valid for Friedmann cosmology but have no obvious relevance to MOND cosmology. But it is proven below that MOND on small scale is consistent with Friedmann on large scale.

3. A critical length scale for Modified Dynamics

I now consider the meaning of the initial radius r_i in eqs. 12 and 13. Looking back at the Newtonian expression eq. 3 we see that, at any epoch characterized by some value of density and pressure, the acceleration of the radius of the shell increases linearly with r . This implies that there should be some critical radius r_c , beyond which the acceleration exceeds the MOND acceleration a_o and the dynamics is Newtonian. That is to say, on all scales greater than r_c , the usual Newtonian expressions, eqs. 3 and 5 apply to the expansion of a spherical region and the evolution of the scale factor.

This critical length scale is given by

$$r_c = \sqrt{\frac{GM}{a_o}} \quad (14)$$

where again M is the active gravitational mass. Making use of eqs. 4-7, eq. 14 becomes

$$r_c = a_o \left| \frac{\Omega_o H_o^2}{2x^3} + \frac{\Omega_r H_o^2}{x^4} - \lambda H_o^2 \right|^{-1} \quad (15a)$$

or

$$r_c = \frac{2a_o}{\Omega_o H_o^2} x^3 \quad (15b)$$

when the universe is matter dominated,

$$r_c = \frac{a_o}{\Omega_r H_o^2} x^4 \quad (15c)$$

when the universe is radiation dominated, and

$$r_c = \frac{a_o}{\lambda H_o^2} \quad (15d)$$

when the universe is vacuum energy dominated. Therefore, at any epoch characterized by a scale factor x , modified dynamics can only apply in regions smaller than r_c . This critical radius grows faster than both the scale factor and the horizon, l_h , in the radiation- and matter-dominated regimes (i.e., $r_c/l_h \propto t^2$).

In Fig. 1 r_c is plotted against scale factor. Here and elsewhere below the cosmological parameters are taken to be $H_o = 75$ km/(s Mpc), $\Omega_o = 0.02$, $\Omega_r = 4.48 \times 10^{-5}$ and $a_o = 1.2 \times 10^{-8}$ cm/s². This value of Ω_o is consistent with the baryonic content of the Universe implied by considerations of primordial nucleosynthesis (Walker et al. 1991) and with estimates of the stellar mass in galaxies and intra-cluster hot gas (Carlberg et al. 1998); in the context of MOND this would be the total matter content of the Universe (i.e., no significant contribution from non-baryonic dark matter). The density parameter in radiation, Ω_r , is exactly that for a black body of 2.73 K and the assumed Hubble parameter. The value of the acceleration parameter, a_o , is that determined from fitting to the extended rotation curves of nearby galaxies (Begeman et al. 1990). For the purposes of this plot the cosmological constant has been set to zero. The age of a model universe with this assumed H_o and Ω_o would be 1.26×10^{10} years which is consistent with the recent determinations of the ages of globular clusters in light of the new cluster distance scale (Chaboyer et al. 1997)

Also shown in Fig. 1 is the horizon scale ($\approx ct$) as a function of scale factor determined by numerical integration of eq. 5. It is evident that at early epochs r_c is much smaller than the horizon size. This suggests that any causally connected region of the Universe can be isotropic and homogeneous with the expansion governed by the usual Friedmann equation, eq. 5. That is to say, for spatial separations larger r_c , it is valid to apply a Universal scale factor and, presumably, the RW metric. However, about a typical point in the Universe there exist a smaller volume with radius r_c within which modified dynamics and eq. 12

applies; i.e., in which separations cannot be described in terms of a scale factor and which expand at a slower rate than the universe at large. Thus at any epoch inhomogeneities must be present on a scales smaller than r_c . For a universal scale factor greater than 0.23, the critical MOND radius r_c exceeds the horizon scale. This means that the entire causally connected Universe has become MONDIAN at a redshift of about 3.3 and can no longer be described by the RW metric.

This interpretation does contain a logical conundrum. In an actual MOND universe with Friedmann expansion on a horizon scale but slower MOND expansion on sub-horizon scales, not every point in the fluid can possibly be a center of MOND-dominated expansion and collapse; it is not possible that a horizon-scale volume can expand while smaller spherical regions about every point within that volume re-collapse. This is a paradox of the present non-relativistic treatment which can only be resolved in a more complete theory—a theory involving fluctuations in which local peaks probably play the role of seeds for MOND-dominated expansion and re-collapse with voids developing elsewhere. But, in any case, is likely that r_c is the approximate scale below which there exist MOND-induced inhomogeneities at any epoch in an evolving Universe.

Accepting this interpretation, we note that larger and larger masses enter the MOND regime as the universe evolves. Given that r_o is the comoving scale of mass M_c and taking $x = r_c/r_o$, we have

$$M_c = \frac{\Omega_o H_o^2}{2G} \left(\frac{r_c}{x}\right)^3 \quad (16)$$

This is also shown in Fig. 1 where it is evident that objects of galaxy mass ($10^{11} M_\odot$) enter the MOND regime at $x \approx 7 \times 10^{-3}$ corresponding to a redshift of 140.

Obviously then the appropriate value for the initial radius, r_i in eq. 13a, would be the radius at which eq. 12 first applies to the expansion of the spherical region; i.e.,

$$r_i = r_c. \quad (17)$$

This would be about 14 kpc for the galaxy size region. Moreover, the initial expansion velocity would be given by the Hubble expansion on a scale of r_i :

$$u_i = H r_i = r_i H_o \sqrt{\Omega_o (r_o/r_i)^3 + \Omega_r (r_o/r_i)^4 + (1 - \Omega_o) (r_o/r_i)^2} \quad (18)$$

which is about 320 km/s for the the $10^{11} M_\odot$ region. We then find, from eq. 13, that $q^2 = 1.25$ and $r_m/r_i = 3.5$; i.e., after entering the MOND regime at a redshift in excess of 100, a galaxy mass would only expand by a factor of about four before re-collapsing. The time-scale for this expansion is given by

$$\Delta t = \sqrt{\pi} q H^{-1} \left(\frac{r_m}{r_i}\right) \operatorname{erf}(q) \quad (19)$$

(Felten 1984); i.e., objects of galaxy size and smaller enter the MOND regime early and fall out of Hubble expansion on a time-scale comparable to the age of the Universe at that epoch. For the $10^{11} M_\odot$ sphere this would be approximately 3×10^8 years. Re-collapse and virialization might take three or four times longer, so we see that with modified dynamics in the context of a low density Friedmann universe, galaxies should be in place as virialized objects when the Universe is about 10^9 years old corresponding to a redshift of 9 or 10.

4. The early radiation dominated Universe

In the early universe the size of spherical regions dominated by modified dynamics is small compared to the horizon. Taking eq. 15c and noting that the temperature of the black body radiation scales with the inverse of the scale factor, we have

$$r_c = \frac{a_o}{\Omega_r H_o^2} \left(\frac{T_o}{T} \right)^4 \quad (20)$$

which, for the assumed cosmology becomes

$$r_c = 4.54 \times 10^{31} \left(\frac{T_o}{T} \right)^4 \text{ cm} \quad (21)$$

With $T_o = 2.73$ K and $T = 10^9$ K, corresponding to the epoch of nucleosynthesis, we find $r_c = 2.5 \times 10^{-3}$ cm. The scale of the horizon in the radiation-dominated era is given by

$$l_h \approx 0.5(\Omega_r H_o^2)^{-\frac{1}{2}} \left(\frac{T_o}{T} \right)^2 c \quad (22)$$

which is 10^{13} cm at the epoch of nucleosynthesis. That is to say, the expansion of the Universe as a whole is identical to that of the standard Big Bang with the scale of modified dynamics being 15 orders of magnitude smaller than the horizon size.

But because nucleosynthesis occurs on a smaller scale still— that of internucleon spacing ($\approx 10^{-7}$ cm)— we must consider the fate of these small regions of non-standard dynamics. Taking eqs. 4 and 9 but now with the equation of state for radiation, we find

$$\ddot{r} = -(\Omega_r H_o^2 a_o r_o^4)^{\frac{1}{2}} r^{-\frac{3}{2}} \quad (23)$$

which integrates to

$$\dot{r}^2 = u_i^2 - 4(\Omega_r H_o^2 a_o r_o^4)^{\frac{1}{2}} \left[\frac{1}{r_i^{\frac{1}{2}}} - \frac{1}{r^{\frac{1}{2}}} \right] \quad (24a)$$

Given eq. 18 for u_i and that $x = r_c/r_o$ we find with eq. 15c that

$$u_i^2 = (\Omega_r H_o^2 a_o^2 r_o^4)^{\frac{1}{3}}. \quad (24b)$$

Then it is straight forward to show that, in the absence of other considerations, any such region will expand more slowly than the universe as a whole and will re-collapse after expanding by a factor $r_m/r_i = 16/9$.

This might well seem devastating not just for primordial nucleosynthesis, but also for the overall homogeneity of the early universe. There is, however, another physical effect which must be considered. The slower expansion of these regions on the scale of r_c will very rapidly lead to density and hence pressure gradients which will resist re-collapse. Because the gravitational acceleration in these regions is so small, only small density gradients are required to keep these regions expanding with the Universe at large. The gravitational acceleration in these small MOND regions is (by definition) on the order of a_o . So the pressure gradient required to resist re-collapse can be estimated from

$$\frac{1}{\rho} \frac{dp}{dr} = a_o \quad (25)$$

Setting $p = \frac{1}{3}\rho c^2$ and $dr = r_c$ and making use of eq. 20 we estimate the corresponding density fluctuation over this scale to be

$$\frac{\delta\rho}{\rho} = \frac{3a_o^2}{\Omega_r H_o^2 c^2} \left(\frac{T_o}{T}\right)^4 \quad (26)$$

which implies $\delta\rho/\rho \approx 10^{-31}$ when $T = 10^9$ K. In the early radiation dominated universe, modified dynamics results in no significant deviation from homogeneity and the thermal history is identical to that of the standard Big Bang. This means that all of the results on nucleosynthesis in the standard model carry over to a MOND cosmology.

At the epoch of recombination ($T = 4000$ K), where radiation still dominates in low Ω_o models, we find $\delta\rho/\rho \approx 4 \times 10^{-10}$. That is to say, the MOND-induced inhomogeneities would be five orders of magnitude less than the density fluctuations implied by the observed fluctuations in the CMB.

The argument on pressure gradients resisting MOND collapse can be re-framed in terms of the critical Jeans mass for gravitational instability. The Jeans mass is, effectively, identical to the virial mass which, in the context of MOND, is given by

$$M_J = \frac{9}{G a_o} c_s^4 \quad (27)$$

where c_s is the sound speed in the fluid being considered (Milgrom 1989). Before decoupling of matter and radiation, $c_s = c/\sqrt{3}$ implying that $M_J \approx c^4/G a_o$ which, given that $a_o \approx c H_o$, is on the order of the total mass of the present observable Universe. Obviously this is vastly greater than the mass enclosed in a MOND region (at the epoch of nucleosynthesis this is

approximately 10^{-12} g); MOND dominated gravitational collapse is clearly an impossibility before hydrogen recombination.

After recombination, the Jeans mass of the baryonic component becomes

$$M_J = \frac{9}{G a_o} \left(\frac{k T_m}{m} \right)^2 \quad (28)$$

where k is the Boltzmann constant, m is the mean atomic mass, and T_m is the temperature of the matter. But collapse can still not occur until the Jeans mass falls below the critical mass in a MOND dominated region. From eqs. 16 and 20 this is found to be

$$M_c = \frac{a_o^3 \Omega_o H_o^2}{2G(\Omega_r H_o^2)^3} \left(\frac{T_o}{T} \right)^9 \quad (29)$$

in the radiation-dominated regime. We see from eqs. 28 and 29 that while the Jeans mass decreases with the square of the radiation temperature, the critical MOND mass rapidly increases as the temperature falls. This is illustrated in Fig. 2 after the epoch of recombination ($T_{rec} \approx 4000$ K) assuming that $T_m = T^2/T_{rec}$ (true for non-relativistic mono-atomic fluid). The MOND critical mass becomes comparable to the Jeans mass (eqs. 27 and 28) when the radiation temperature has fallen to

$$T = \left[\frac{1}{18} \frac{\Omega_o H_o^2}{(\Omega_r H_o^2)^3} \left(\frac{m}{k} \right)^2 a_o^4 T_o^9 T_{rec}^2 \right]^{\frac{1}{13}}, \quad (30)$$

with the cosmology assumed above this is 2.5×10^3 K, or somewhat later than the epoch of recombination (this value is obviously quite insensitive to the actual values of the cosmological parameters). The corresponding value of the Jeans mass and critical MOND mass is about $10^5 M_\odot$. This means that shortly after recombination pressure gradients are no longer effective in preventing MOND-induced collapse. However, it is argued below that $M_J < M_c$ is a necessary but not a sufficient condition for MOND-dominated expansion and collapse.

5. The formation of structure

Expansion of a low density universe ($\Omega_o \ll 1$) will remain radiation-dominated until well after recombination; for the cosmology assumed here this occurs at $x = 4.48 \times 10^{-3}$ ($z = 222$). It is clear from Fig. 2 that regions having a mass less than $4 \times 10^9 M_\odot$ enter the MOND regime while radiation still dominates the energy density of the Universe. Taken at face value then, it would seem that one should apply eq. 24 to the MOND-dominated expansion of regions above the Jeans mass which enter the MOND regime in the period

between recombination and matter-radiation density equality. However, during this period the horizon is much larger than the scale over which MOND applies (at matter-radiation equality the horizon is 4 Mpc but the scale of modified dynamics is only 3 kpc). After recombination, the photons are uncoupled to the matter, so it would be impossible for a MOND region to re-collapse while the passive gravitational mass in radiation still dominates the gravitational deceleration; the photons free-stream to the horizon. MOND-dominated expansion and re-collapse as described by eq. 12 would begin for all masses between $300 M_\odot$ (the Jean’s mass) and $4 \times 10^9 M_\odot$ (the critical MOND mass) at the epoch of matter-radiation equality (indicated by the heavy solid line in Fig. 2).

The actual mass in a MOND-dominated region at the epoch of matter-radiation equality, M_e , is extremely sensitive to the cosmological parameters. This may be determined from eqs. 15 and 16 (setting 15b equal to 15c) and is found to be

$$M_e = \frac{32a_o^3\Omega_r^6}{G\Omega_o^8H_o^4} \quad (31)$$

Combining with eq. 7 we find

$$M_e = 3.7 \times 10^9 \left[\left(\frac{\Omega_o}{0.02} \right) \left(\frac{H_o}{75} \right)^2 \right]^8 M_\odot \quad (32)$$

Matching the observed light element abundances with the predictions of primordial nucleosynthesis (Walker et al. 1991), implies that $0.018 < \Omega_o(H_o/75)^2 < 0.027$. Then with eq. 32 we find that $4.3 \times 10^8 M_\odot < M_e < 3.7 \times 10^{10} M_\odot$.

It is interesting that the mass scale over which MOND applies at the epoch of matter-radiation equality— when MOND collapse can begin and significant inhomogeneities can form— corresponds to that of low to moderate mass galaxies. Perhaps this offers some explanation for the fact that the lowest-mass virialized building-blocks of the Universe are galaxies (the existing globular clusters and dwarf galaxies re-collapsed simultaneously and may have survived due to incomplete merging). But in any case, for objects of any mass scale, the separation of from the Hubble expansion and subsequent re-collapse can be described by eq. 12 (the Felten equation) with the initial radius being the critical MOND radius given by eq. 15 for objects of mass greater than $10^9 M_\odot$, and the scaled comoving radius ($r = xr_o$) at the epoch of matter-radiation equality for lower mass regions.

This dynamical history is shown for various mass scales in Fig. 3 which is a plot of the scale factor (r/r_o) of regions of different mass as a function of cosmic time to the point of maximum expansion. These curves are determined from numerical integrations of eq. 12. The cosmic scale factor corresponding to Friedmann expansion (eq. 5) is also shown (again the cosmological term has been set to zero). The vertical dotted line shows the epoch of matter-radiation equality for this particular cosmological model.

It is evident that objects of globular cluster mass ($10^5 M_\odot$) re-collapse very soon after matter-radiation equality; maximum expansion is reached at a cosmic time of 2.3×10^7 years corresponding to a redshift of 156. Massive galaxies ($10^{11} M_\odot$) reach this point of maximum expansion at $t = 3 \times 10^8$ years or $z = 26$. Clusters of galaxies ($10^{14} M_\odot$) begin to re-collapse at 2.65×10^9 years ($z=3$).

The mass which is just turning around at the present epoch is $3.7 \times 10^{15} M_\odot$. The comoving scale is 66 Mpc but the present radius would be 29 Mpc. This would correspond to the mass and scale of superclusters as noted by Felten (1984). Taken literally the implication would be that a region of 30 Mpc about a typical observer should be collapsing rather than expanding, which is evidently not the case locally. This result, which might be taken as an argument against a pure MOND cosmology, neglects likely complications arising in a real Universe filled with significant density enhancements and peculiar accelerations. In the fully-developed MOND universe at the present epoch, the large scale inhomogeneities and resulting tides will most likely cause large aspherical distortions of the developing structure. Thus, the effects of distant matter cannot be ignored; i.e., the fundamental assumption underlying this treatment of isolated regions breaks down. Given the enhanced tidal effects and the fact that, in MOND, the internal dynamics of a region is effected by the external acceleration field, the “external field effect” (Milgrom 1983a), the growth of pancakes, filaments and voids would seem natural. The present turnaround radius of 30 Mpc may only give an estimate of the scale of structure which has significantly separated out of the present Hubble flow, as suggested by Felten (1984).

Since the entire present Universe is MONDIAN (in the absence of a dynamically significant cosmological constant), then on all scales out to the horizon, the expansion cannot be uniform; the mean value of the Hubble parameter grows with scale. This also implies that the mean density of matter within a spherical region should decrease out to the horizon. This may be determined by integrating eq. 12 for spheres with comoving radii larger than 66 Mpc (corresponding to the current shell which is just turning around at 29 Mpc) out to the horizon scale of 4000 Mpc. The result is shown in Fig. 4 which is a log-log plot of the ratio of the mean density inside a finite spherical volume to the mean density of the Universe as a function the present radius of the sphere (not the comoving radius). It is evident that the average density smoothly increases from the horizon down to a scale of 30 Mpc where it is 10 times larger than the mean density.

The literal and naive interpretation of Fig. 4, and that which would seem most consistent with the treatment of isolated spherical regions, is that this would represent the density distribution about a single observer in a MOND Universe. But if we require, consistent with the Cosmological Principle, that the observer have no special position, then

an equally valid interpretation is that the average density distribution about any observer in the MOND universe declines smoothly to the horizon– that the matter distribution is non-analytic (fractal) and does not imply a special position for one observer (Coleman & Pietronero 1991).

It has been claimed, from analysis of redshift catalogues, that the mean density of galaxies about a given galaxy does decrease with scale (eg. Coleman & Pietronero 1991) out to cosmological distances. Although this claim is controversial (Peebles 1993), it is generally consistent with expectations for a pure MOND cosmology. The actual radius of cross-over to homogeneity would depend upon the value of a possible cosmological constant. With a cosmological term large enough to be dynamically significant ($\lambda \approx 1$), the critical radius for modified dynamics (eq. 15a), after first becoming infinitely large, asymptotically approaches a constant value: $r_c \rightarrow (1/6)(c/H_o)$, i.e., about 1/6 the current horizon if $H_o \approx 75$ km/s-Mpc (the observed value of a_o would then correspond to $cH_o/6$). In this case we might expect large scale homogeneity and uniform exponential expansion of the Universe on sub-horizon scales larger than several hundred Mpc.

6. Conclusions

Although there is not yet a plausible candidate for a general theory of gravity which predicts the MOND phenomenology in the limit of low accelerations, consideration of the dynamics of a finite spherical volume may contain elements of a realistic MOND cosmology. Perhaps the most interesting conclusion that can be drawn from such a exercise is that modified dynamics over finite separations is compatible with Friedmann cosmology on large scale; until relatively recently in the history of the Universe MOND could not dominate the dynamical evolution of the Universe in general. At earlier epochs, the scale over which MOND applies, r_c (within which the deceleration of Hubble expansion is less than a_o), is smaller than the size of a causally connected region which implies that the Universe as a whole can be isotropic and adequately described by the RW metric with expansion governed by the usual Friedmann equation. However, the fact that the expansion is slower in MOND-dominated regions implies that inhomogeneities must be present at any epoch on a scale of r_c and smaller. In the early radiation dominated Universe, the magnitude of these MOND-induced inhomogeneities is very small ($\delta\rho/\rho \approx 10^{-31}$ when $T=10^9$ K), because of the small pressure gradients required to restore uniform expansion. Therefore, the thermal and dynamical history of the early MOND Universe is exactly that of the standard Big Bang and all predictions relevant to the nucleosynthesis of the light elements carry over to MOND cosmology.

After non-relativistic matter dominates the mass density of the Universe (which can be rather late in a low density Universe), MOND cosmology diverges from that of standard cosmology. At the epoch of matter-radiation equality, objects with mass up to $4 \times 10^9 M_\odot$ rapidly collapse to form virialized objects. The fact that this mass scale, which is the mass in the MOND regime at radiation-matter equality, is comparable to that of low mass galaxies seems significant: objects of this mass would be the principal virialized building blocks in the Universe. Moreover, this mass scale emerges naturally from the basic dynamics; astrophysical considerations such as cooling vs. collapse time scales do not play a role. Although objects of smaller mass (down to $10^2 M_\odot$) collapse and virialize first, a process probably accompanied by star formation, these objects would rapidly merge in the the larger collapsing regions. This suggests that galaxy formation is primarily dissipationless; that the stellar content of galaxies may be in place before the galaxies actually form. Of course, early star formation could be limited by processes such as photo-dissociation of H_2 as in standard scenarios (Haiman et al. 1997); these self-limiting processes could keep much of the matter content of the universe in gaseous form as seems to be implied by the observations of rich clusters.

Many of these low mass galaxies galaxies would merge to form more massive objects as larger and larger scales come into the MOND regime. A spherical region with the mass of a large galaxy ($10^{11} M_\odot$) reaches maximum expansion and begins to re-collapse at a redshift of 26 which implies that large galaxies should be in place as virialized objects by redshift of 5 to 10. This is earlier than the epoch of galaxy formation in the standard CDM paradigm (Frenk et al. 1988). Moreover, from Fig. 3 it is evident that regions with the size and mass of a cluster have reached maximum expansion by a cosmic age of 2.7×10^9 years corresponding to a redshift of three. This means that by $z=3$ not only do massive galaxies exist but they are also significantly clustered (the density of the $10^{14} M_\odot$ region would be enhanced by a factor of 6.5 over the mean at this redshift). This may be relevant to the observation of luminous galaxies at $z=3$ which is remarkable not only because they are there but also because of the apparent degree of clustering (Steidel et al. 1997). Such observations may be able to distinguish between the cosmogony sketched here and that of the standard CDM paradigm.

The largest objects being virialized now would be clusters of galaxies with masses in excess of $10^{14} M_\odot$. Superclusters would only now be reaching maximum expansion. Such a scenario of structure formation is hierarchical in the extreme and as such bears a resemblance to the more standard (CDM) scenarios of the build up of structure. But here, dissipationless dark matter is not required to enhance structure formation; structure forms inevitably on the MOND scale of r_c because of the effective logarithmic potential.

In the present Universe regions approaching the horizon scale would be subject to a scale-dependent deceleration due to modified dynamics. This would lead to a Universe in which the mean run of density about a galaxy decreases smoothly to a cosmic scale. The actual scale for approach to homogeneity would depend upon the value of the cosmological constant; for $\lambda \approx 1$ corresponding to a zero curvature Universe, the density and expansion of the Universe would be more or less uniform on scales greater than several hundred Mpc. In such a Universe, the Hubble parameter would also be, on average, scale dependent, increasing with the separation between objects out to some significant fraction of the Hubble radius. However, these are the aspects of MOND cosmology which most depend upon the unknown properties of the underlying theory—such as the value of the cosmological constant and whether or not MOND phenomenology saturates at some lower value of acceleration and attraction returns to inverse square.

The details of cosmology and cosmogony sketched here are dependent upon the assumptions which underly this procedure: no effects of the surrounding universe on the finite spherical volume, no return to Newtonian dynamics at lower accelerations, no variation of a_o with cosmic time. A different set of assumptions is also plausible and would lead to a different MOND cosmology. The remarkable aspect of the cosmology resulting from these assumptions is the fact that the pre-recombination dynamical and thermal evolution is identical to that of the standard Big Bang. But in any case, it seems inevitable that in a MOND cosmology, structure formation proceeds much more rapidly and efficiently than in standard cosmologies due to the effective logarithmic potential.

Friedman expansion on horizon scale combined with modified dynamics on smaller scale suggests that density peaks may be required to play the role of seeds or centers about which MOND-dominated expansion and re-collapse occurs. Apart from this, primordial density fluctuations have played no role in the present discussion of structure formation. In the correct relativistic theory this will probably not be the case; for example, in stratified quadratic scalar-tensor theory (Sanders 1997), there are no effects of modified dynamics in the absence of scalar field gradients; in a perfectly homogeneous Universe, the metric is RW and structure never develops. This suggests that in a proper theory fluctuations may be essential for the development of structure which may then proceed, qualitatively, as described above.

One should be cautious about pushing these results too far. In standard theory, the Newtonian dynamics of an expanding region takes on cosmological significance only in retrospect, that is, after the application of General Relativity and the construction of a relativistic cosmology. Here, the order is reversed— the rules of MOND are applied to a finite spherical region before the development of the appropriate relativistic theory. But

it is possible that many of the aspects of a fully relativistic cosmology may be previewed, at least in a qualitative sense, by such an exercise. While this remains to be seen, it is of considerable interest that the resulting cosmology does seem to reconcile the extreme homogeneity of the early radiation-dominated Universe with early galaxy formation and the extreme range of structure observed in the matter-dominated era. Moreover, the fact that this can be accomplished without the necessity of invoking hypothetical non-baryonic dark matter is entirely consistent with the original motivation for modified dynamics as an alternative to dark matter on the scale of galaxies and galaxy clusters.

I am very grateful to J.D. Bekenstein and M. Milgrom for helpful comments on this work.

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Fig. 1.— A log-log plot of the evolution of the horizon, l_h (solid line), and the radius within which MOND applies, r_c (dotted line), both in Mpc, as a function of cosmic scale factor for the given cosmological parameters ($H_o = 75$ km/s-Mpc, $\Omega_o = 0.02$). Also shown is the mass enclosed within r_c (dashed line) in units of $10^{11} M_\odot$. The vertical long dashed line indicates the epoch at which matter and radiation contribute equally to the deceleration. It is evident that when the scale factor is smaller than 0.23 ($z > 3.3$) the region over which modified dynamics applies is smaller than the size of a causally connected region.

Fig. 2.— A log-log plot of the critical mass within a MOND-dominated (solid line) and the Jeans mass in the context of MOND (dotted line), both in M_\odot , as a function of cosmic scale factor, for the adopted cosmological parameters. The epochs of recombination and matter-radiation equality are shown by the vertical dashed lines. MOND-induced collapse proceeds rapidly after matter-radiation equality with initially all masses between the Jeans mass ($334 M_\odot$) and the critical MOND mass ($3.7 \times 10^9 M_\odot$) separating out of the Hubble flow and re-collapsing. This range is indicated by the heavier solid line.

Fig. 3.— A log-log plot of the scale factor for regions of various mass (the radius over the comoving radius) as a function of cosmic time, for the adopted cosmological parameters. The MOND cosmogony hierarchical and bottom-up, with small structures forming soon after matter-radiation equality (indicated by the vertical dotted line). Massive galaxies ($10^{11} M_\odot$) separate out of the Hubble flow early and reach maximum expansion at a cosmic age of 3×10^8 years. Supercluster mass structures ($10^{16} M_\odot$) are only now reaching maximum expansion and turn-around.

Fig. 4.— A log-log plot of the mean density, at the present epoch, inside spherical shells as a function of radius in Mpc. The density is given in terms of the mean universal density. It is evident that the density decreases smoothly to the horizon implying that a MOND-dominated Universe, at present, would be very inhomogeneous. The scale for approach to homogeneity is smaller if the cosmological constant is not zero (≈ 600 Mpc for $\lambda \approx 1$).







